In the following module, we will show how we can use curve fitting (commonly referred to as regression) to extract an important atmospheric property called the optical depth. This property will be defined below.

Imagine the following scenario. A radiation detection device is aimed at the sun and monitors the radiation as seen from the ground.

Consider the radiation along path 1. This path occurs if the sun is directly overhead (this never occurs in NYC but only along the equator). What should be the energy seen by the detector? We can imagine the energy as a stream of photons. If the atmosphere were empty, there would be nothing to stop the photons from reaching the detector and the flux on the ground would be the same as the flux at the top of the atmosphere. However, the atmosphere is filled with gas composed of a variety of molecules (Oxygen, Nitrogen, Carbon Dioxide, Water Vapor etc) as well as small particles (Dust, Salts, Soot etc). All of these objects will obstruct the photons from reaching the ground either by absorbing the photon or scattering the photon away from the light path. In our discussions, we will not make any distinction between absorption and scattering since both processes effectively cause the light photon to miss the detector. We call the combined process of scattering and absorption extinction.

\[ \text{Extinction} = \text{Absorption} + \text{Scattering} \]

Loosely speaking, the more particles in the atmosphere, the more extinction occurs and the less energy reaches the detector. How can we quantify this process?
Consider the following scenario.

\[ P \downarrow \alpha(z) \downarrow dz \downarrow P+dP \]

A beam with incident power \( P \) hits an atmospheric layer with an extinction coefficient per unit length given by \( \alpha \). We define this coefficient to be the probability that the layer (if the layer width is 1) extinguishes a photon. Therefore, the number of photons gained is \( dN = -\alpha N \). From this result, we have \( \frac{dP}{P} = -\alpha(z)dz \) since **Power and Flux is proportional to the number of photons**.

This is a differential equation whose solution is simply \( I_G = I_H e^{-\int_0^G \alpha(z)dz} \) where \( H \) is the atmosphere top and \( G \) is the ground. If we define the following quantity called the optical depth as \( \tau_{opt} = \int_0^Z \alpha(z)dz' \), the ground flux can be written as \( I_G = I_H e^{-\tau} \) (*).

where \( \tau \) is the total atmospheric optical depth. The optical depth parameter is a measure of how difficult it is for light to penetrate the atmosphere. If the optical depth is small, the light penetrates easily. If the optical depth is very large \( \tau >> 1 \), the atmosphere extinguishes the light. To determine \( \tau \), we may try to use the (*) equation. This equation however, requires knowledge of \( I_H \) which is not easy to obtain. To obtain more information, we can ask what would happen if we took a sun measurement at an angle different than directly over head (i.e. at a different time). If we assume that the atmosphere extinction depends only on altitude and not on horizontal location, we have \( I_G(\theta) = I_H e^{-\tau \sec(\theta)} \). If we take multiple measurements, we could in theory solve for the coefficients \( I_H \) and \( \tau \) which fit the data in a Least Squared sense. However, in this model the input – output relation is not linear and therefore harder to deal with. To overcome this problem, we can take logarithms of both sides resulting in

\[ \log(I_G(\theta)) = \log(I_H) - \tau \sec(\theta) \]

This relation describes a straight line if we make the following definitions.

\[ y = \log(I_G) \]
\[ y_0 = \log(I_H) \]
\[ m = -\tau \]
\[ x = \sec(\theta) \]
Therefore, if we fit the transformed \( \{x_i, y_i\} \) data to a line, the optical depth is given as 
\[
\tau = -a.
\]

Until now, we have not considered the wavelength of light. We can imagine that some particles extinct light at different wavelengths with different efficiencies. For example, many atmospheric molecules (for example, CO\(_2\)) absorb light very efficiently if the wavelength is just right and if the wavelength is off by even a little amount, the absorption is much weaker. Therefore, we could imagine that if we would plot optical depth verses wavelength, a spike would occur at that particular absorbing wavelength. The height of this spike could then be used to determine how many absorbing molecules of that type occur and the wavelength of the spike would be a signature of which molecule is absorbing.

Even in the case of non-absorbing particles, the wavelength dependence of the extinction can give us information concerning the size of the scattering particles since particles at different sizes can scatter light differently at a given wavelength. Therefore, the frequency information of atmospheric extinction can be used to get some idea of the number, size and type of particles in the atmosphere. However, the procedures used to deduce such information from the data is quite complicated and will not be discussed here.

With the discussion above, we now have a procedure to obtain total atmospheric optical depth as a function of wavelength given that the measurements \( I_G(\theta_i, \lambda_j) \) are available. Here \( j = 1: N_{\text{wav}} \) is a wavelength index and \( i = 1: N_{\text{ang}} \) is the angle index.

The data is stored in a matrix whose dimensions are \( N_{\text{ang}} \times N_{\text{wav}} \)

For each wavelength \( j = 1: N_{\text{wav}} \)

1. Transform the measurement data
   \[
   x_i = \sec(\theta_i), \quad y_i = \log(I_G(\theta_i) \lambda_j), \quad i = 1: N_{\text{ang}}
   \]

2. Perform a linear fit of the data \( \{x_i, y_i\} \) to obtain \( \tau = -a \)

3. Store this optical depth in the \( j^{th} \) position of a vector

Finally, plot the optical depth as a function of wavelength.
Assignment Procedure

1. Load the Sun-Photometer Data into MATLAB. The files that store the Detector Intensities are data1.mat (July 23), data4.mat (July 13) and data6.mat (July 16). The wavelengths are stored in a vector in file wavelengths

\textit{load wave} (2024 x 1) vector containing the wavelengths in nm

\textit{load data1} (Nang x 2024) matrix of the Intensity of light at the ground for all 2024 wavelengths and Nang different angles for the first day.

\textit{load data4} (Nang x 2024) \textit{load data1} (Nang x 2024) matrix of the Intensity of light at the ground for all 2024 wavelengths and Nang different angles for the second day.

\textit{load data6} (Nang x 2024) \textit{load data1} (Nang x 2024) matrix of the Intensity of light at the ground for all 2024 wavelengths and Nang different angles for the third day.

\textit{load data1ang} (Nang x 1) vector storing the values of the observation angles for data1

\textit{load data4ang} (Nang x 1) vector storing the values of the observation angles for data4

\textit{load data6ang} (Nang x 1) vector storing the values of the observation angles for data6

2. Plot the data and the best fit regression line for 6 different wavelengths. Make sure your choice of wavelengths does not include include any absorption resonances (give the wavelengths)

3. Determine the optical Depth as a function of wavelength for the 3 different days and plot the results. Which day is the dirtiest? Even if there is no aerosols or particles, the normal atmosphere has a certain standard molecular optical depth. An empirical formula for this molecular optical depth is

\[
\tau_{mol}(\lambda) = c_1 \left( \frac{\lambda}{\lambda_0} \right)^{-4} \left[ 1 + c_2 \left( \frac{\lambda}{\lambda_0} \right)^{-2} + c_3 \left( \frac{\lambda}{\lambda_0} \right)^{-4} \right]
\]

where \( \lambda \) is measured in nanometers, \( \lambda_0 = 500 \text{nm} \), \( c_1 = 0.09364 \), \( c_2 = 0.0374 \) and \( c_3 = 0.00142 \). Superimpose the optical depths obtained in part 2 with the molecular optical depth. What happens if the measured optical depth is less than the molecular value. Plot the true aerosol optical depth (total-molecular) versus wavelength.

4. On your optical depth plot, superimpose the optical depth with the smallest and largest values, the optical depth can have (within reason).